**Module 3: Trigonometric Identities, Inverse Functions, and Applications**

**IV. Area and Solution of Triangles: Law of Sines and Law of Cosines**

After completing this section, you should be able to:

* find the area of a triangle, given the length of two sides and the sine of the included angle
* state the law of sines and the law of cosines
* apply the law of sines and the law of cosines to solve oblique triangles

**A. Area of a Triangle**

Recall that the area of area of a triangle is ½ *base × height*. For a right triangle, the lengths of the legs are the base and height, so it is easy to calculate the area.

What happens if a triangle is an **oblique triangle**, a triangle that is not a right triangle? How can you determine the area? You will see that trigonometry will be helpful!

Consider an oblique triangle *ABC*, whose angles have measures *A*, *B*, and *C*, and whose corresponding opposite sides have lengths *a*, *b*, and *c*. The goal of this topic is to investigate how trigonometry can be used to find the area and to solve this type of triangle.

|  |  |
| --- | --- |
|  |  |
| One type of oblique triangle is an **acute triangle**, a triangle whose three angles are all acute. | A second type of oblique triangle is an **obtuse triangle**, a triangle that contains an obtuse angle, which is an angle whose measure is between 90° and 180°.  Since the sum of the angles of a triangle is 180 degrees, it is not possible for a triangle to contain more than one obtuse angle. |

Denote the area of a triangle by *K*. Consider each type of triangle separately.

**Case 1: Acute Triangle**

|  |  |
| --- | --- |
| Draw the altitude from *C* to the opposite side of the triangle. (Recall that an altitude is line segment connecting a vertex to the opposite side, so that the line segment is perpendicular to the opposite side.)  Label the point of intersection *D*.  The length of the altitude is the height *h* of the triangle, and the base *AB* has length *c*. |  |

|  |
| --- |
| Now there are two right triangles formed, *CAD* and *CDB*. Trigonometric ratios can now be computed for the acute angles *A* and *B*:  , so *h* = *b*sin *A*.  Area of *ABC* = *K* = ½ *base* × *height* = ½ *ch*= ½ *c*(*b* sin *A*)*=*½*bc*sin*A*.  Alternatively, , so *h* = *a* sin *B*.  Area of *ABC* = *K* = ½ *base* *x height*= ½ *ch*= ½ *c*(*a* sin *B*)*=*½*ac*sin*B*.  (By drawing an altitude from *A* to the opposite side of length *a*, it could also be shown that *h* = *b* sin *C* and that the area is½*ab*sin*C*.)  To summarize, the area *K* of the triangle is one half the product of the lengths of any two sides and the sine of the angle formed by those sides (that angle is called the **included angle**). |

**Case 2: Obtuse Triangle**

|  |  |
| --- | --- |
| Suppose the obtuse angle has measure *A*. For convenience, assume that *A* is measured in degrees.  The height *h* of the triangle is the length of the line segment from *A* perpendicular to the line containing the base of the triangle. Label the point of intersection *D*.  There are two right triangles, *CDA* and *CDB*.  *CDA* has an acute angle of measure 180° – *A*. |  |
| , so *h* = *b* sin (180° – *A*).  Note that the obtuse angle *A* is a quadrant II angle whose reference angle is 180° – *A*. (See module 2, topic II-B.)  Since the sine is positive in quadrant II, sin *A* = sin (180° – *A*).  So *h* = *b* sin (180° – *A*) = *b* sin *A*, just as in case 1. | |

Also, , so *h* = *a* sin *B*, just as in case 1.

Therefore, the same area formulas that were true for acute triangles also hold for any oblique triangle.

In fact, it is easy to check that these formulas also work in the case of a right triangle.

**The Area of a Triangle**

The area *K* of a triangle is one half the product of the lengths of any two sides and the sine of the included angle.

If the triangle has angles of measure *A*, *B*, and *C*, and the corresponding sides opposite the angles have lengths *a*, *b*, and *c*, then

|  |
| --- |
|  |

**Example IV.A.1:**Find the area of an equilateral triangle whose sides have lengths of 8 centimeters.

**Solution:**

Each angle of an equilateral triangle *ABC* has measure 60°. Since each side has length 8, *a* = *b* = *c* = 8.

|  |  |
| --- | --- |
| Area of *ABC* | = 1/2 *bc* sin *A*  = 1/2 (8)(8) sin 60° =  =  ≈ 27.7 square centimeters |

The area of the triangle is  cm2, which is approximately 27.7 cm2.

**B. The Law of Sines and Solution of Triangles: AAS, ASA, and SSA Cases**

|  |  |
| --- | --- |
|  |  |

The investigation of oblique triangles revealed that *a* sin *B* and *b* sin *A* are both equal to the height *h*, so we can see that

|  |  |
| --- | --- |
| *a* sin *B* = *b* sin *A* |  |
| = | Divide both sides of the equation by sin *A* sin *B*. |
| = | Simplify. |
| By interchanging the roles of *A* and *C* in the investigation, it can also be shown that | |
|  |  |

Since  and , it is concluded that .

This result relating the lengths of the sides and the sines of the angles is known as the law of sines.

**The Law of Sines**

If a triangle has angles of measure *A*, *B*, and *C*, and the corresponding sides opposite the angles have lengths *a*, *b*, and *c*, then



The law of sines is helpful in solving oblique triangles.

In module 2, you learned how to solve a right triangle. Given a right triangle, if you know the measure of one of the acute angles and the length of a side, or if you know the lengths of two sides, you can solve the triangle. Since a right triangle contains a 90° angle, you actually are given three pieces of information at the outset: the measure of two angles (one of them 90°) and the length of one side, or the measure of one angle (90°) and the length of two sides.

Similarly, in order to solve an oblique triangle, three pieces of information must be provided.

**Example IV.B.1:** Solve the triangle *DEF* if *D* = 42°, *E* = 60° and *d* = 12.

Solution:

Use the information given to sketch and label a triangle.

|  |  |
| --- | --- |
|  | Known: Angles *D* and *E* and side *d* (two angles and a side opposite one of the known angles)  To find: Angle *F* and sides *e* and *f* |

Since the sum of the measures of the three angles is 180°,

|  |  |
| --- | --- |
| *D* + *E* + *F* | = 180° |
| 42° + 60° + *F* | = 180° |
| *F* | = 78° |

Use the law of sines to find *e* and *f*:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  | | --- | --- | --- | |  | = |  | |  | = | Substitute. | | *e* | = | Solve for *e*. | | *e* | ≈ 16 | Calculate. | | |  |  |  | | --- | --- | --- | |  | = |  | |  | = | Substitute. | | *f* | = | Solve for *f*. | | *f* | ≈ 18 | Calculate. | |

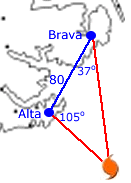
*D* = 42°, *E* = 60°, *F* = 78°, *d* = 12, *e* ≈ 16, and *f* ≈ 18.

In the example, the given information consisted of the measure of two angles (*D* and *E*) and the length *d* of the side opposite angle *D*. This is an illustration of a case known as AAS. AAS denotes a situation for which you are given the measures of two angles (A and A) and the length of a side (S) opposite one of the angles. Provided the sum of the measures of the two given angles is less than 180°, the AAS case has a unique solution that can be found by using the law of sines. If the sum of the measures of the two given angles is equal to or greater than 180°, then there is no possible triangle containing the angles, and therefore no solution.

Try another example.

**Example IV.B.2:**

A satellite image shows a hurricane threatening coastal towns Alta and Brava. The distance between Alta and Brava is 80 miles. A triangle having vertices at Alta, Brava, and the hurricane yields the following information: The angle at Alta is 105° and the angle at Brava is 37°. How far is the hurricane from Alta? How far is the hurricane from Brava?



Solution:

Label the triangle to indicate what is known and what is being sought. For convenience, let Alta be denoted by *A* and Brava by *B*.

|  |  |
| --- | --- |
|  | The distance between Alta and the hurricane is *b*.  The distance between Brava and the hurricane is *a*.  Known: Angles *A* and *B*, and side *c* (two angles and the included side.)  To find: Side *a* and side *b*. |

In order to determine *a* and *b*, it is helpful to find the measure of the third angle *C*. Since the sum of the measures of the three angles is 180°,

|  |  |  |
| --- | --- | --- |
| *A* + *B* + *C* | = 180° |  |
| 105° + 37° + *C* | = 180° | Substitute. |
| *C* | = 38° | Solve for *C*. |

Now both *c* and *C* are known.

Use the law of sines to find *a* and *b*:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  | | --- | --- | --- | |  | = |  | |  | = | Substitute. | | *a* | = | Solve for *a*. | | *a* | ≈ 126 | Calculate. | | |  |  |  | | --- | --- | --- | |  | = |  | |  | = | Substitute. | | *b* | = | Solve for *b*. | | *b* | ≈ 78 | Calculate. | |

**Conclusion:** The hurricane is approximately 78 miles from Alta and approximately 126 miles from Brava.

In this example, the given information consisted of the measure of two angles (*A* and *B*) and the length *c* of the side common to both angles (the included side). This is an illustration of the ASA case. ASA denotes a situation for which you are given the measures of two angles (A and A) and the length (S) of the included side. Provided the sum of the measures of the two given angles is less than 180°, the ASA case has a unique solution that can be found by using the law of sines. If the sum of the measures of the two given angles is larger than 180°, then there is no possible triangle containing the angles, and therefore no solution.

So far, you have seen that you can solve any triangle if you know the measures of two of the angles and the length of any side. These are the AAS and ASA cases.

Now suppose you know the lengths of two sides and the measure of an angle opposite one of the sides. This is the SSA case.

**Example IV.B.3:** Solve the triangle *KMP* if *K* = 30°, *k* = 10, and *m* = 24.

Solution:

Use the information given to sketch and label a triangle.

|  |  |
| --- | --- |
|  | Known: Sides *k* and *m*, and angle *K* (two sides and an angle opposite one of the sides, SSA)  To find: Side *p*, and angles *M* and *P* |

Use the law of sines to find *M*:

|  |  |  |
| --- | --- | --- |
|  | = |  |
|  | = | Substitute. |
|  | = | Take reciprocals. |
| sin *M* | = | Solve for sin *M*. |
| sin *M* | = 1.2 | Calculate. |

There is no solution. The sine value can never be larger than 1. There is no angle for which the sine is larger than 1. No triangle has an angle 30, opposite side of length 10, and adjacent side of length 24.

So, it is possible that an SSA case has no solution. The following example illustrates another SSA case.

**Example IV.B.4:** Solve the triangle *ABC* if *A* = 37°, *a* = 7, and *c* = 10.

Solution:

Use the information given to sketch and label a triangle.

|  |  |
| --- | --- |
|  | Known: Sides *a* and *c*, and angle *A* (two sides and an angle opposite one of the sides, SSA)  To find: Side *b*, and angles *B* and *C* |

Use the law of sines to find *C*:

|  |  |  |
| --- | --- | --- |
|  | = |  |
|  | = | Substitute. |
|  | = | Start by taking reciprocals. |
| sin *C* | = | Multiply both sides by 10. |
| sin *C* | ≈ 0.8597 | Calculate. |

Since sine is positive in quadrants I and II, this trigonometric equation has two solutions:

*C* ≈ 59° or *C* ≈ 180° – 59° = 121°.

This leads to two possible triangles meeting the given specifications. One triangle is acute and the other triangle is obtuse.

|  |  |
| --- | --- |
| **Case 1:** *C* ≈ 59°    Since the sum of the measures of the three angles is 180°, | **Case 2:** *C* ≈ 121°        Since the sum of the measures of the three angles is 180°, |
| |  |  |  | | --- | --- | --- | | *A* + *B* + *C* | = 180° |  | | 37° + *B* + 59° | = 180° | Substitute. | | *B* | = 84° | Solve for *B*. | | Use the law of sines to find *b*: | | | |  | = |  | |  | = | Substitute. | | *b* | = | Solve for *b*. | | *b* | ≈ 12 | Calculate. | | |  |  |  | | --- | --- | --- | | *A* + *B* + *C* | = 180° |  | | 37° + *B* + 121° | = 180° | Substitute. | | *B* | = 22° | Solve for *B*. | | Use the law of sines to find *b*: | | | |  | = |  | |  | = | Substitute. | | *b* | = | Solve for *b*. | | *b* | ≈ 4.4 | Calculate. | |

There are two triangles that satisfy the conditions provided:

**Solution 1:** *A* = 37°, *B* ≈ 84°, *C* ≈ 59°, *a* = 7, *b* ≈ 12, and *c* = 10.

**Solution 2:** *A* = 37°, *B* ≈ 22°, *C* ≈ 121°, *a* = 7, *b* ≈ 4.4, and *c* = 10.

The previous two examples illustrate SSA cases which have either no solution or two solutions. There is one more type of SSA case, which is considered in the following example.

**Example IV.B.5:** Solve the triangle *ABC* if *A* = 117°, *a* = 6.70, and *b* = 1.50.

Solution:

Use the information given to sketch and label a triangle.

|  |  |
| --- | --- |
|  | Known: Sides *a* and *b*, and angle *A* (two sides and an angle opposite one of the sides, SSA)  To find: Side *c*, and angles *B* and *C* |

Use the law of sines to find *B*:

|  |  |  |
| --- | --- | --- |
|  | = |  |
|  | = | Substitute. |
|  | = | Take reciprocals. |
| sin *B* | = | Solve for sin *B*. |
| sin *B* | ≈ 0.1995 | Calculate. |

Since sine is positive in quadrants I and II, this trigonometric equation has two solutions:

*B* ≈ 12° or *B* ≈ 180° – 12° ≈ 168°.

However, 168° can be discarded as a possibility, because that would make the sum of the angle measures too large: *A* + *B* = 117° + 168° = 285°, which is greater than 180°.

Therefore, the only possibility is *B* ≈ 12°.

Since the sum of the measures of the three angles is 180°,

|  |  |  |
| --- | --- | --- |
| *A* + *B* + *C* | = 180° |  |
| 117° + 12° + *C* | ≈ 180° | Substitute. |
| *C* | ≈ 51° | Solve for *C*. |

Use the law of sines to find *c*:

|  |  |  |
| --- | --- | --- |
|  | = |  |
|  | = | Substitute. |
| *c* | = | Solve for *c*. |
| *c* | ≈ 5.8 | Calculate. |

*A* = 117°, *B* ≈ 12° , *C* ≈ 51°, *a* = 6.7, *b* = 1.5, and *c* ≈ 5.8. There is exactly one solution.

Examples IV.B.3, IV.B.4, and IV.B.5 all illustrate the SSA case, in which the given information consists of the lengths of two sides and the measure of an angle opposite one of the given sides. An SSA case has either no solution, one solution, or two solutions. Since there can be more than one solution in some instances, the SSA case is often called the *ambiguous case*.

**C. The Law of Cosines and Solution of Triangles: SAS and SSS Cases**

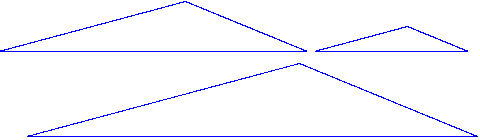
In order to solve an oblique triangle, three pieces of information have been provided. So far, several cases have been considered:

|  |  |
| --- | --- |
| **Case** | **Information Provided** |
| AAS | Two angles and a side opposite one of the angles |
| ASA | Two angles and the included side |
| SSA | Two sides and an angle opposite one of the sides |

There are several other cases not yet studied:

|  |  |
| --- | --- |
| **Case** | **Information Provided** |
| AAA | Three angles |
| SAS | Two sides and the included angle |
| SSS | Three sides |

Recall that two triangles are similar if the measures of their corresponding angles are equal. Therefore, given the measures of three angles, there are infinitely many similar triangles containing those three angles.



The three angles determine the shape of the triangle, but not the size. So, the AAA case is not fruitful; it does not result in a finite number of solutions.

The remaining cases, SAS and SSS, are most easily solved by using a property known as the law of cosines.

**Law of Cosines**

If a triangle has angles of measure *A*, *B*, and *C*, and the corresponding sides opposite the angles have length *a*, *b*, and *c*, then

*a*2 = *b*2 + *c*2 – 2*bc* cos *A*  
*b*2 = *a*2 + *c*2 – 2*ac* cos *B*  
*c*2 = *a*2 + *b*2 – 2*ab* cos *C*

Notice that the law of cosines is a generalization of the Pythagorean theorem:

If one of the measure of one of the angles *A* is 90°, then cos *A* = 0, and

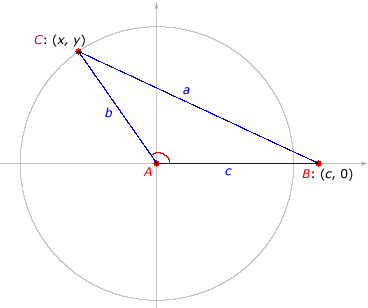
*a*2 = *b*2 + *c*2 – 2*bc* cos *A*  
    = *b*2 + *c*2.

Here is a proof of the law of cosines.

**Proof:**

The equation *a*2 = *b*2 + *c*2 – 2*bc* cos *A* will be established first.

Place the triangle *ABC* so that one of the angles (say *A*) is placed in standard position and the vertex at angle *B* has coordinates (*c*, 0). The side of length *c* lies on the positive *x*-axis.



Denote the coordinates of the vertex at angle *C* by (*x*, *y*). The point is on the terminal side of angle *A* in standard position. Since the length of the side opposite angle *B* is *b*, the point (*x*, *y*) is on the circle of radius *b* centered at the origin. Then cos *A* = *x*/*b* and sin *A* = *y*/*b*. (See module 2, topic II-B.)

Therefore *x* = *b* cos *A* and *y* = *b* sin *A*.

The vertex at angle *C* has coordinates (*x*, *y*) = (*b* cos *A*, *b* sin *A*), the vertex at angle *B* has coordinates (*c*, 0), and the distance between them is *a*, the length of the side opposite angle *A*.

Applying the distance formula,

|  |  |  |
| --- | --- | --- |
| *a*2 | = (*x* – *c*)2 + (*y* – 0)2 | Find the square of the distance between (*x*,*y*) and (*c*, 0). |
|  | = (*b* cos *A* – *c*)2 + (*b* sin *A* – 0)2 | Substitute for *x* and *y*. |
|  | = *b*2 cos2 *A* – 2*bc* cos *A* + *c*2 +*b*2 sin2*A* | Square the binomials. |
|  | = *b*2(cos2 *A* + sin2 *A*) + *c*2 – 2*bc* cos *A* | Regroup. |
|  | = *b*2 + *c*2 – 2*bc* cos *A* | Apply the identity sin2 *A* + cos2 *A* = 1. |

Now one of the three equations has been established. To establish that *b*2 = *a*2 + *c*2 – 2*ac* cos *B*, place the angle of measure *B*in standard position and carry out the same procedure. To establish that *c*2 = *a*2 + *b*2 – 2*ab* cos *C*, place the angle of measure *C* in standard position and carry out the same procedure.

The following examples show how the law of cosines can be applied to SAS and SSS cases.

**Example IV.C.1:** Solve the triangle *ABC* if *B* = 48°, *a* = 6.0, and *c* = 7.2.

Solution:

Use the information given to sketch and label a triangle.

|  |  |
| --- | --- |
|  | Known: Sides *a* and *c*, angle *B* (two sides and the included angle, SAS)  To find: Side *b*, and angles *A* and *C* |

Use the law of cosines to find *b*:

|  |  |  |
| --- | --- | --- |
| *b*2 | = *a*2 + *c*2 – 2*ac* cos *B* |  |
|  | = 6.02 + 7.22 – 2(6.0)(7.2) cos 48° | Substitute. |
|  | ≈ 36.00 + 51.84 – 57.81 | Calculate. |
|  | ≈ 30.03 | Simplify. |
| *b* | ≈ 5.5 | Take the square root of both sides. |

To find another angle, the law of sines or the law of cosines can be used. If the law of sines is used, since the sine is positive in quadrant I and in quadrant II, care must be taken to consider the possibility of several solutions. If the law of cosines is used, since cosine is positive in quadrant I and negative in quadrant II, there is only one case to consider.

Using the law of cosines to find *A*:

|  |  |  |
| --- | --- | --- |
| *a*2 | = *b*2 + *c*2 – 2*bc* cos *A* |  |
| 6.02 | = 5.52 + 7.22 – 2(5.5)(7.2) cos *A* | Substitute. |
| 36.00 | = 30.25 + 51.84 – 79.2 cos *A* | Calculate. |
| –46.09 | = –79.2 cos *A* | Simplify. |
| 0.5819 | = cos *A* |  |

Since cos *A* is positive and angle *A* must be less than 180, the equation cos *A* = 0.5819 has only one solution.

|  |  |  |
| --- | --- | --- |
| *A* | ≈ 54° | Use a calculator to find the angle whose cosine is 0.5819. |

Since the sum of the measures of the three angles is 180°,

|  |  |  |
| --- | --- | --- |
| *A* + *B* + *C* | = 180° |  |
| 54° + 48° + *C* | ≈ 180° | Substitute the approximation of *A* and the given value of *B*. |
| *C* | ≈ 78° | Solve for *C*. |

*A* ≈ 54°, *B* = 48°, *C* ≈ 78°, *a* = 6.0, *b* ≈ 5.5, and *c* = 7.2.

**Note:** In this example, angle *A* was found by using the law of cosines, and then angle *C* was found by using the fact that the sum of the angle measures is 180°. If instead, angle *C* was found by using the law of cosines, and then *A* was determined, the results would also be correct. However, the values might be slightly different, due to the errors introduced by rounding numbers.

**Example IV.C.2:** A real-estate property in a commercial district is priced at $8.25 per square foot. If the property has a triangular shape, measuring 210 feet by 170 feet by 150 feet, what is the cost?

Solution:

The shape of the property is an oblique triangle. To find the cost of the property, calculate the area of the property in square feet, and then multiply by the cost per square foot.

The area formula for an oblique triangle uses the lengths of two sides and the sine of the included angle. The given information consists of the lengths of three sides. Use the techniques for solving triangles in order to find the measure of one of the angles.

Use the information given to sketch and label a triangle.

|  |  |
| --- | --- |
|  | Known: Sides *a*, *b*, and *c* (three sides, SSS)  To find: One angle, say *A* |

Use the law of cosines to find *A*:

|  |  |  |
| --- | --- | --- |
| *a*2 | = *b*2 + *c*2 – 2*bc* cos *A* |  |
| 2102 | = 1702 + 1502 – 2(170)(150) cos *A* | Substitute. |
| 44,100 | = 28,900 + 22,500 – 51,000 cos *A* | Calculate. |
| –7,300 | = –51,000 cos *A* | Simplify. |
| 0.1431 | = cos *A* |  |

Since cos *A* is positive and angle *A* must be less than 180°, the equation cos *A* = 0.1431 has only one solution.

|  |  |  |
| --- | --- | --- |
| *A* | ≈ 82° | Use a calculator to find the angle whose cosine is 0.1431. |
|  | | |

|  |  |
| --- | --- |
| Area of *ABC* | *=*½ *bc* sin *A* |
|  | = ½ (170)(150) sin 82° |
|  | ≈ 12,600 square feet |
|  | |

|  |  |
| --- | --- |
| Cost of property | = Area × Cost per square foot |
|  | = (12,600 square feet) × ($8.25 per square foot) |
|  | = $104,000, to the nearest thousand. |

The cost of the property is $104,000, to the nearest thousand.

The previous example illustrated an SSS case. We needed to determine just one of the angles to solve the word problem. To solve the triangle, it would be necessary to find all three angles. The second angle could also be determined by using the law of cosines. The third angle can be determined by computing 180° minus the sum of the measures of the first two angles.

For the task of solving a triangle, the following table summarizes the cases studied and the properties to apply.

|  |  |  |
| --- | --- | --- |
| **Case** | **Information Provided** | **Property to Apply** |
| AAS | Two angles and a side opposite one of the angles | Law of sines |
| ASA | Two angles and the included side | Law of sines |
| SSA | Two sides and an angle opposite one of the sides | Law of sines; leads to possibilities of no solution, one solution, or two solutions |
| AAA | Three angles | None; cannot determine the sides |
| SAS | Two sides and the included angle | Law of cosines |
| SSS | Three sides | Law of cosines |

The SSA case is the only case which may produce more than one solution.

[*Return to top of page*](https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/Area%20and%20Solution%20of%20Triangles%20-%20Law%20of%20Sines%20and%20Law%20of%20Cosines.html#pagetop)

[**Report broken links or any other problems on this page.**](http://help.umuc.edu/)  
  
[**Copyright © by University of Maryland University College.**](https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/common/copyright.html)